Meadow Vale Primary School

Calculation Policy

This calculation policy has been written to provide staff and parents with the guidance they will need in order to assist children with the appropriate mental and written calculation strategies they are using in class.

At Meadow Vale, we use the 'White Rose Hub' format as a basis for our planning. White Rose follows the **Concrete – Pictorial – Abstract** approach to teaching maths.

- **Concrete** is the 'doing' stage, using concrete objects to solve problems.

- The **pictorial** or 'seeing' stage uses representations of objects to solve problems. This helps children make the connection between the physical object and abstract levels of understanding, which is the stage they move onto next.

- The **abstract** stage brings in mathematical symbols, for example +, -, x, ÷ to indicate addition, subtraction, multiplication and division.

The emphasis, initially, is on mental calculations skills, but progresses to the written strategies that children are expected to become fluent with. Whilst this document has been organised into the expected outcomes for each year group, it is important to recognise that children develop their mathematical skills at different rates and learning should be pitched at a level that is suitable to them.

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EYFS

Long Term Plan

In EYFS, our aim is to provide the children with opportunities to practise and improve their skills in counting numbers, calculating simple addition and subtraction problems, and to describe shapes, spaces, and measures. We believe children should be exposed to different representations of mathematical concepts in order to embed conceptual understanding. One of the aims under the Characteristics of Effective Learning is 'creating and thinking critically.' Children are encouraged to make links, find new ways to do things, solve problems, change strategies as needed, make predictions and develop ideas of grouping, sequencing, cause and effect.

Early Learning Goal for Numbers:

- Children can count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract 2 single-digit numbers and count on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

Early Learning Goal for Shape, Space and Measure:

- Children use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems.
- They recognise, create and describe patterns.
- They explore characteristics of everyday objects and shapes and use mathematical language to describe them.

Key Mathematical Vocabulary

NUMBER

Number: Zero, number one, two, three … to twenty and beyond teens numbers, eleven, twelve … twenty none how many …? count, count (up) to, count on (from, to), count back (from, to) count in ones, twos, fives, tens is the same as more, less odd, even few pattern pair

Place value: Ones, tens, digit, the same number as, as many as more, larger, bigger, greater fewer, smaller, less, fewest, smallest, least most, biggest, largest, greatest one more, ten more one less, ten less compare order size first, second, third… twentieth last, last but one before, after next between

Estimating: Guess how many …? estimate nearly close to about the same as just over, just under too many, too few enough, not enough

Addition and subtraction: Add, more, and make, sum, total altogether double one more, two more … ten more how many more to make …? how many more is … than …? how much more is …? take away how many are left/left over? how many have gone? one less, two less, ten less … how many fewer is … than …? how much less is …? difference between

Multiplication and division: Sharing doubling halving number patterns

Fractions: Parts of a whole, half, quarter

MEASUREMENT

Measure: size compare guess, estimate enough, not enough too much, too little too many, too few nearly, close to, about the same as just over, just under Length metre length, height, width, depth long, short, tall high, low wide, narrow thick, thin longer, shorter, taller, higher … and so on longest, shortest, tallest, highest … and so on far, near, close

Weigh: Weighs, balances heavy, light heavier than, lighter than heaviest, lightest scales

Capacity and volume: Full, empty, half full, holds, container

Time: Days of the week, Monday, Tuesday … day, week birthday, holiday morning, afternoon, evening, night bedtime, dinner time, playtime today, yesterday, tomorrow before, after next, last now, soon, early, late quick, quicker, quickest, quickly slow, slower, slowest, slowly old, older, oldest new, newer, newest takes longer, takes less time hour, o'clock clock, watch, hands

Money: coin penny, pence, pound price, cost buy, sell spend, spent pay

GEOMETRY

Properties of shape: pattern flat curved, straight round hollow, solid sort make, build, draw size bigger, larger, smaller symmetrical pattern, repeating pattern match 2-D shape corner, side rectangle (including square) circle triangle 3-D shape face, edge, vertex, vertices cube pyramid sphere cone

Position and direction: over, under above, below top, bottom, side on, in outside, inside around in front, behind front, back beside, next to opposite apart between middle, edge corner direction left, right up, down forwards, backwards, sideways

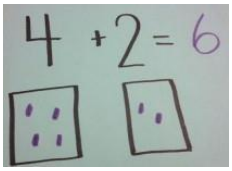
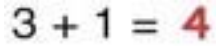
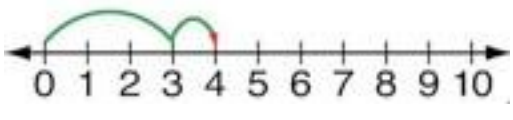
The Counting Principles

The foundations of maths are taught in nursery. These can be organised into the five counting principles.

1. **The one-to-one principle** – this involves children assigning one number name to each object being counted. Children need to ensure they count each object only once.
2. **The stable-order principle** – children need to understand that when counting numbers must be said in a certain order.
3. **The cardinal principle** – children need to understand that the number name assigned to the final object of a group is the total number of objects in that group.
4. **The abstract principle** – this involves children learning that anything can be counted, including things that cannot be touched, e.g. sounds, movements.
5. **The order-irrelevant principle** – this involves children understanding that the order we count a group of objects is irrelevant, there will be still be the same number.

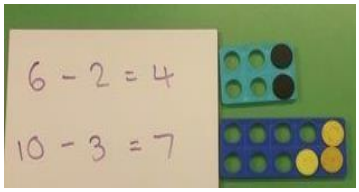
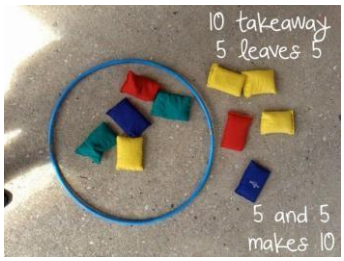
Addition

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE/MODELS AND IMAGES	
Children begin to combine groups of objects or pictures using concrete apparatus.	
Solve simple problems using fingers.	
Construct number sentences verbally or using cards to go with practical activities.	
Children are encouraged to read number sentences aloud in different ways e.g. “Three add two equals 5” “5 is <u>equal</u> to three and two.”	
Count on to find the answer.	
Have an understanding of what “more” means and be able to say what is one more than a given number.	
Number tracks can be introduced to count up on and to find one more.	
Children make a record in pictures, words or symbols of addition activities.	
When appropriate, numicon shapes are introduced to identify 1 more/less, combine pieces to add and find number bonds.	
Number lines can be used alongside number tracks and practical apparatus to solve addition calculations and word problems.	

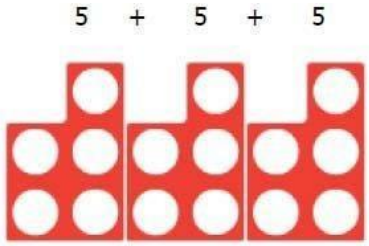
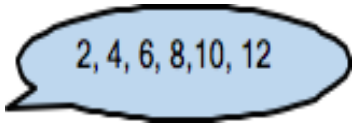
Subtraction

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE/MODELS AND IMAGES	
Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.	
Solve simple problems using fingers.	
Construct number sentences verbally or using cards to go with practical activities.	
Children are encouraged to read sentences aloud in different ways “five subtract one leaves four” “four is equal to five subtract one.”	
Count back to find the answer.	
Have an understanding of what “less” means and be able to say what is one less than a given number. What is 1 less than 9? 1 less than 20?	
Number tracks can be introduced to count back and to find one less.	
Children make a record in pictures, words or symbols of subtraction activities.	
Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems.	

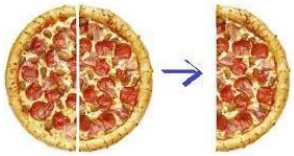
Multiplication

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE/MODELS AND IMAGES	
The link between addition and multiplication can be introduced through doubling and reinforced through repeated addition of the same number.	
Numicon is used to visualise the repeated adding of the same number.	
Children begin with mostly pictorial representations e.g. How many groups of 2 are there? *2 + 2 + 2 + 2 + 2, so 5 groups of 2.*	
Real life contexts and use of practical equipment is used to count in repeated groups of the same size e.g. How many wheels are there altogether?	
Children are encouraged to read number sentences aloud in different ways “five times two makes ten” “ten is equal to five multiplied by two” “ten is the same as five lots of two.”	
Count in twos, fives and tens both aloud and with objects.	
Children are given multiplication problems set in a real life context. Children are encouraged to visualise the problem e.g. How many fingers on two hands? How many sides on three triangles? How many legs on four ducks?	

Division and Fractions

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

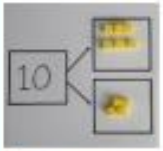
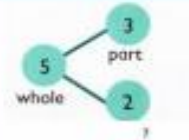
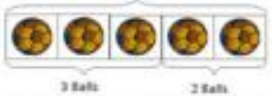
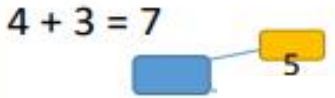
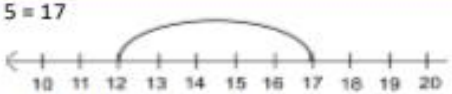
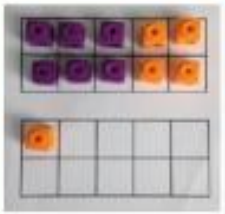
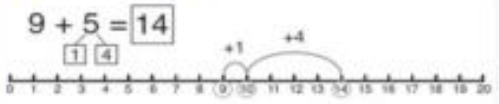
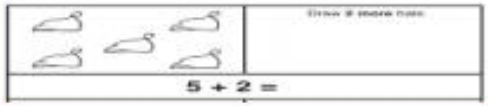
GUIDANCE/MODELS AND IMAGES	
Division can be introduced through halving or sharing an equal amount into 2 groups. Children begin with mostly pictorial representations linked to real life contexts. Children need to see and hear representations of division as both grouping and sharing. <u>Grouping model:</u> Mum has 6 socks. She grouped them into pairs. How many pairs did she make? <u>Sharing model:</u> I have 10 sweets. I want to share them with my friend. How many will we have each? The sharing model is a useful way to introduce young children to fractions e.g. Can you cut the pizza in half? Children make a record in pictures, words or symbols of division activities. Children are encouraged to have a go at recording the calculation that has been carried out e.g. by arranging concrete objects into groups.	    

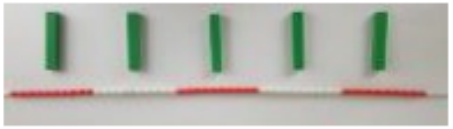
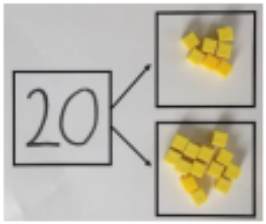
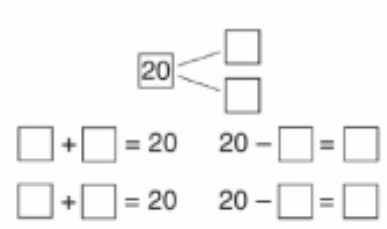
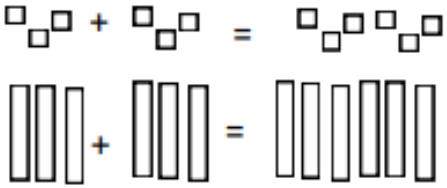
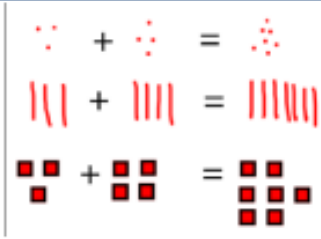
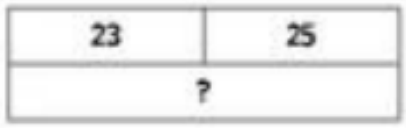
Key Stage 1

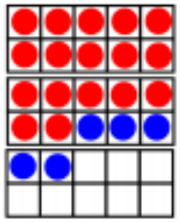
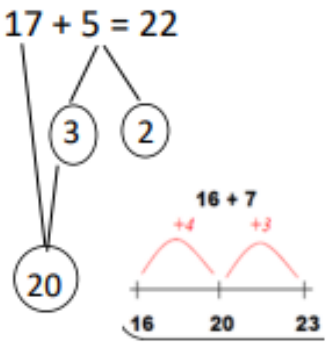
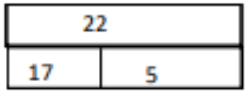
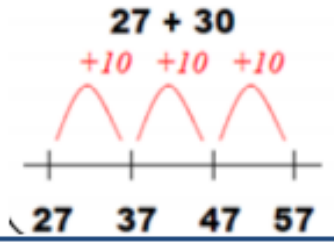
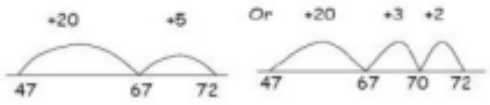
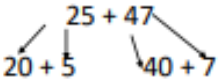
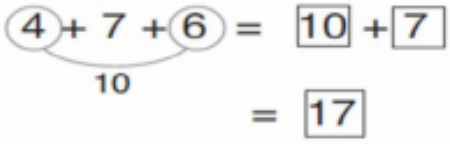
Key Mathematical Vocabulary

Year	Number and Place Value	Addition and Subtraction	Multiplication and Division	Fractions	Measure	Geometry	Data/Statistics
Year 1	Number Counting (on/up/down/back) Before/after More, less, many, few, fewer, fewest, smallest, greater, greatest Equal to, the same as Odd/even Pair Units, ones, tens Ten more/less Digit, numeral Compare, order, size, value Between, halfway between Above/below	Number bonds, number line Add, more, plus, sum, total, altogether Inverse Double, near double Half. Halve Equals, the same as Different between How many more to make...? How many more is… than…? Subtract, take-away, minus How many fewer is… than…? How much less is…?	Odd/even Count in twos, threes, fives Count in tens (forwards from/backwards from) How many times? Lots of, groups of Multiples of, times, multiply, multiply by Repeated addition Array, row, column Double/halve Share, share equally Group in pairs, threes etc. Equal groups of Divide, divided by, left, left over	Whole Equal parts, four equal parts One half, two halves A quarter, two quarters.	Full, half full, empty, holds, container Weigh, balances, heavy, heavier, heaviest, light, lighter, lightest Scales, estimate Time, days of the week – Monday, Tuesday etc. Seasons - winter, spring, summer, autumn, Day, week, month, year, weekend, morning, afternoon, evening, night, midnight, today, yesterday, tomorrow. Hour, o'clock, half-past, clock, hands, watch How long ago...? How often…? Length, width, height, depth Metre, ruler, metre stick. Money, coin, penny, pound, pence, cost, pay, change, total	Position Over, under, above, below Between, middle, edge, centre, corner Direction, journey, across Sort, group Cube, cuboid, pyramid, sphere, cone, cylinder, triangle, square Shape Flat, curved, straight, round. Hollow, solid Corner, point/pointed. Face, side, edge Make, build, draw	
Year 2	Numbers to one hundred Hundreds Partition, recombine Hundred more/less	Carrying Exchanging		Three quarters, one third, a third Equivalence, equivalent	Quarter past/to Metres, kilometres Grams, kilograms Millilitres, litres Temperature, degrees	Rotation, clockwise, anti-clockwise Straight line, ninety degree turn, right-angle Symmetrical, line of symmetry, mirror line, reflection, pattern	Count, tally, sort Vote Graph, Pictogram Represent Table, group, list, set, label, title Most popular, Least popular

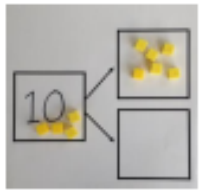
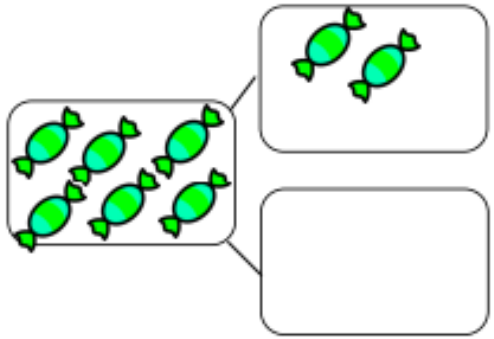
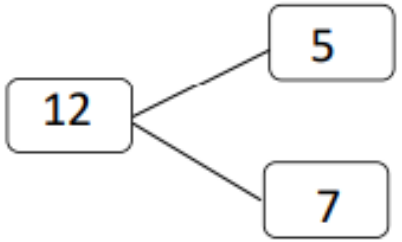
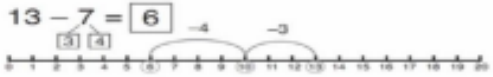
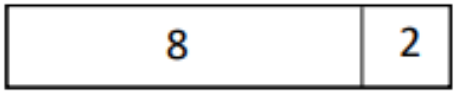
Strategies of teaching: Addition

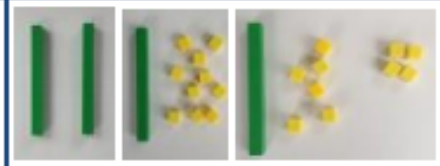
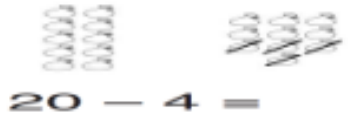
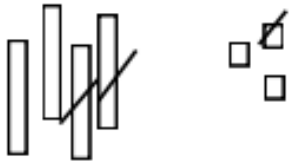
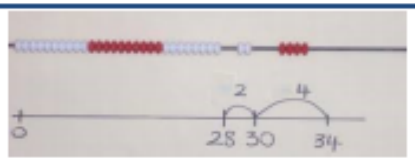
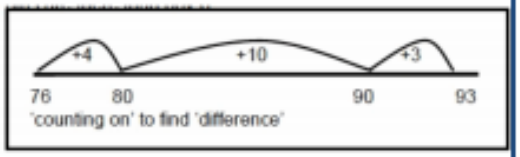
Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	   Use part part whole model. Use cubes to add two numbers together as a group or in a bar. 	    Use pictures to add two numbers together as a group or in a bar.	 10= 6 part ram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 	Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 3 tens + 5 tens = _____ tens 30 + 50 = _____ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part part whole	 Children explore ways of making numbers within 20	 20 □ + □ = 20 20 – □ = □ □ + □ = 20 20 – □ = □	□ + 1 = 16 16 – 1 = □ 1 + □ = 16 16 – □ = 1
Using known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 25 ? 23 + 25 = 48

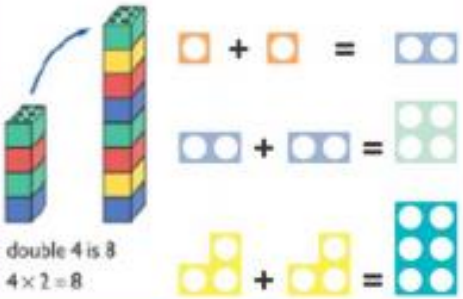
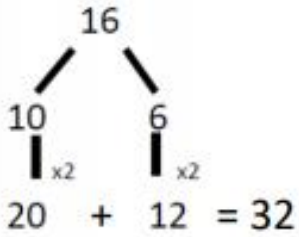
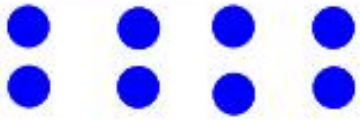
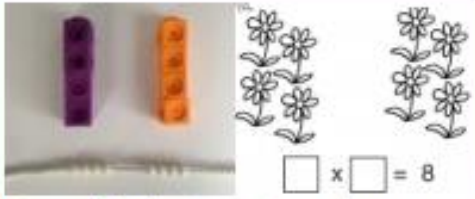
Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	 20 + 40 = 60 5+ 7 =12 60 + 12 = 72
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation. 	 Combine the two numbers that make/ bridge ten then add on the third.

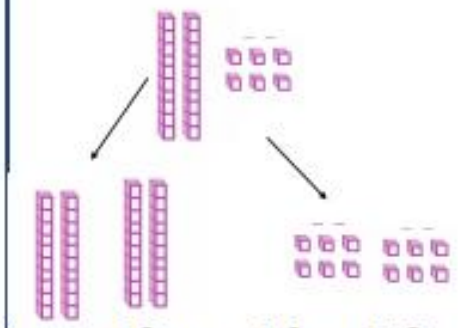
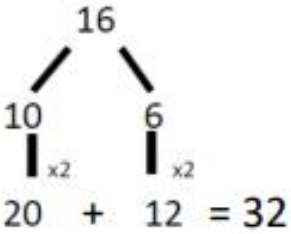
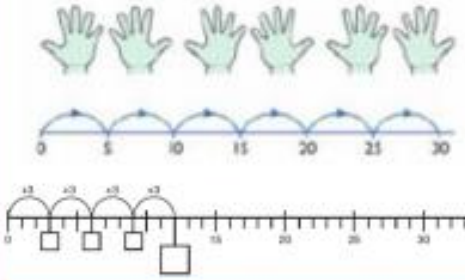
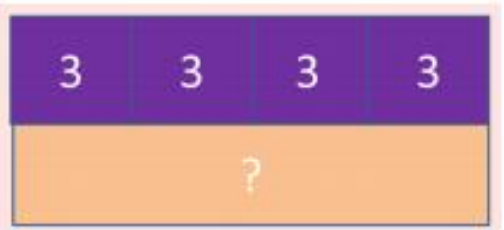
Strategies of teaching: Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

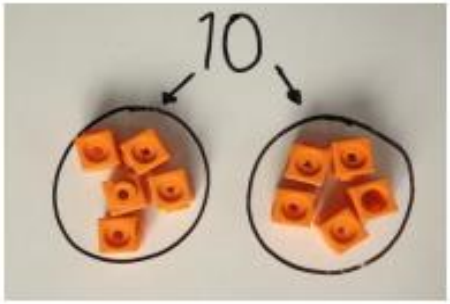
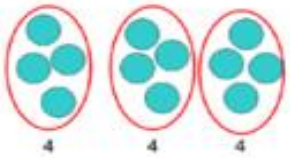
Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'		20—4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.  	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17

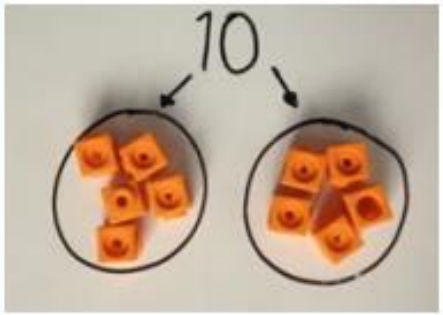
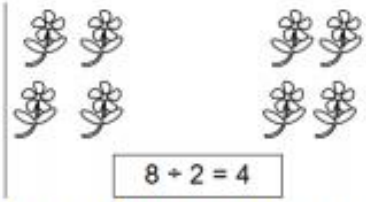
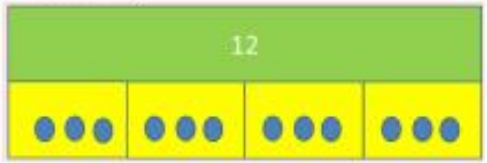
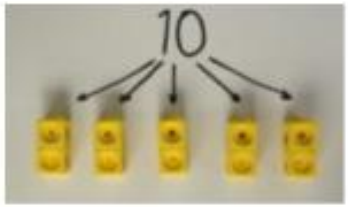
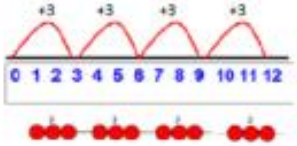
Strategies of teaching: Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers Double 4 is 8 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	 Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  20 + 12 = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40   	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

Strategies of teaching: Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing:  12 shared between 3 is 4	12 shared between 3 is 4

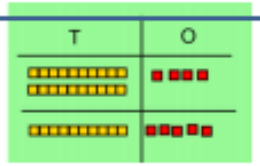
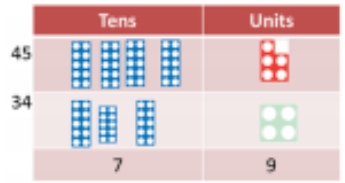
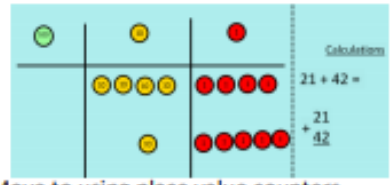
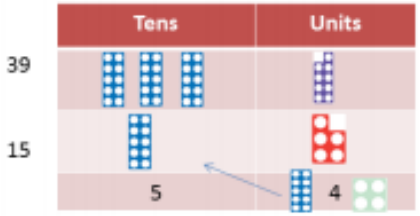
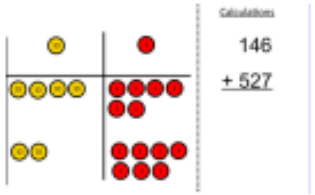
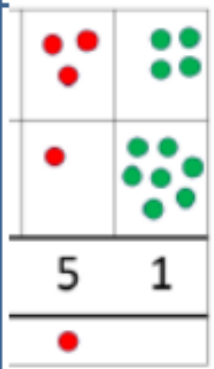
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

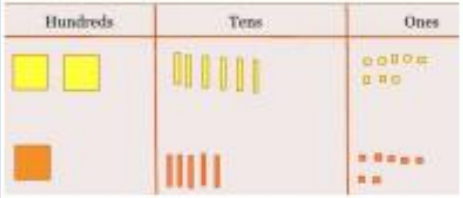
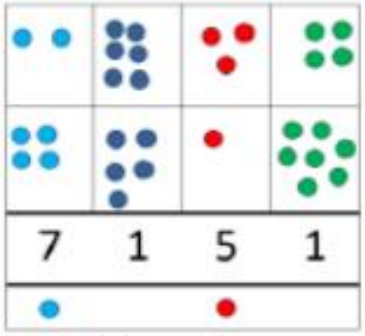
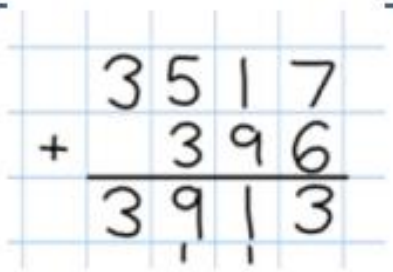
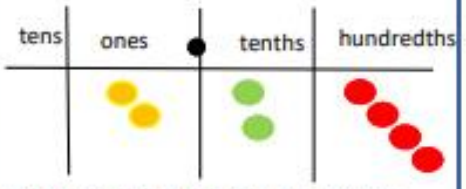
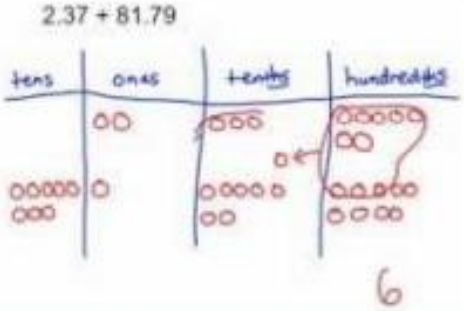
Key Stage 2

Key Mathematical Vocabulary

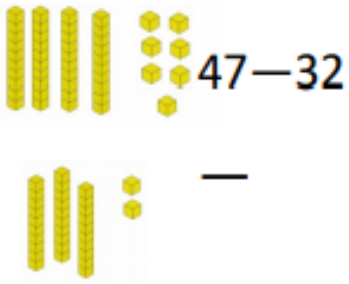
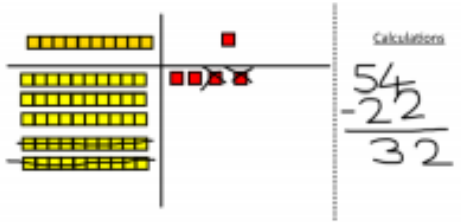
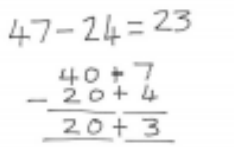
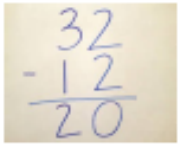
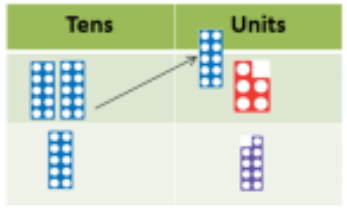
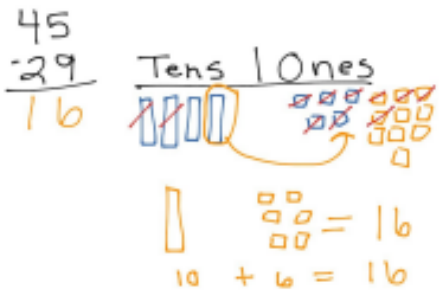
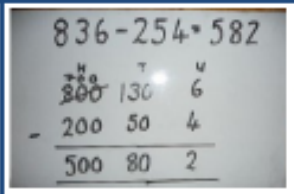
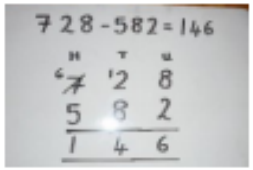
Year	Number and Place Value	Addition and Subtraction	Multiplication and Division	Fractions	Measure	Geometry	Data/Statistics
Year 3	Numbers to one thousand	Column addition Column subtraction	Product Multiples of four, eight, fifty and one hundred Scale up	Numerator, denominator Unit-fraction, non-unit fraction Compare and order tenths	Leap year Twelve hour/24-hour clock Roman Numerals (I to XII)	Greater than, less than ninety degrees Orientation Horizontal, vertical, perpendicular and parallel lines	Chart, bar chart, frequency table, Carroll diagram, Venn diagram Axis, axes Diagram
Year 4	Tenths, hundredths Decimal places Rounding (to nearest) Thousand More/less than Negative integers Count through zero Roman Numerals (I to C)		Multiplication facts (up to 12x 12) Division facts Inverse Derive	Equivalent fractions Half, quarter, one fifth, two fifths, three fifths etc.	Convert, conversion	Coordinates Translation, quadrant x-axis, y-axis Area and perimeter Quadrilaterals Right angle, acute and obtuse angles	Continuous data Line graph
Year 5	Numbers to ten thousand Powers of 10 Symbols		Factor pairs, Composite numbers, Prime numbers, prime factors, square numbers, cubed numbers	Proper fractions, improper fraction, mixed numbers Percentage Decimals	Volume Imperial units Metric units	Regular and irregular polygons Reflex angles Dimensions	
Year 6	Numbers to ten million Linear number sequences Substitute Variables		Common factors Common multiples Long division	Simplify Degree of accuracy		Four quadrants Vertically opposite angles Circumference, radius, diameter	Mean Pie-chart Construct

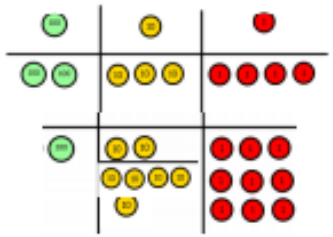
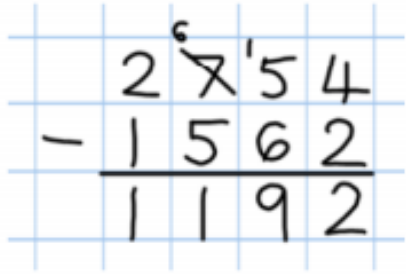
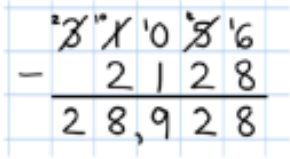
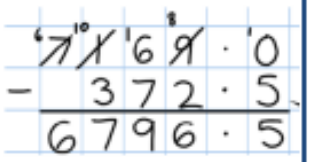
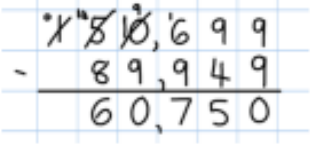
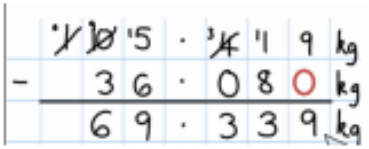
Strategies of teaching: Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers. Y3	 Model using Dienes or numicon Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	2 2 3 + 1 1 4 3 3 7 Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and pv counters. 	 Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	20 + 5 40 + 8 60 + 13 = 73 Start by partitioning the numbers before formal column to show the exchange. 536 + 85 621 1 1

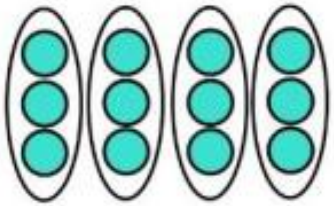
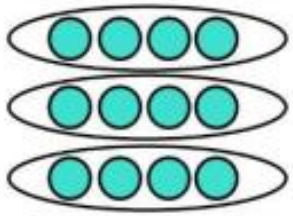
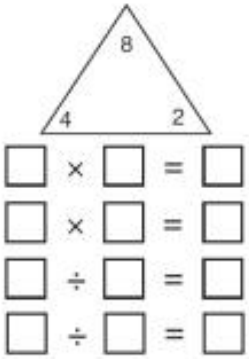
Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	~~Children continue to use dienes or pv~~ counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using pv grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.		72.8 +54.6 127.4 1 1 
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	 Insert zeros for place holders. 

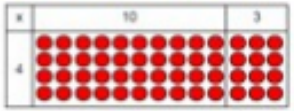
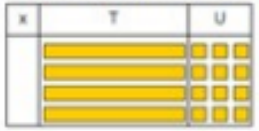
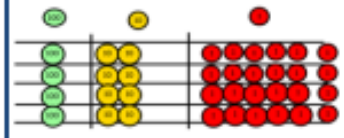
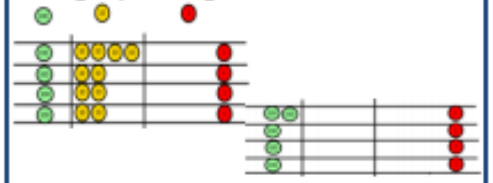
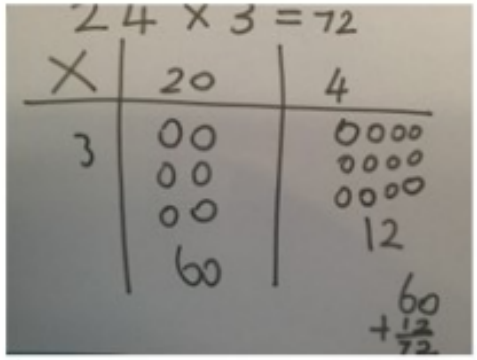
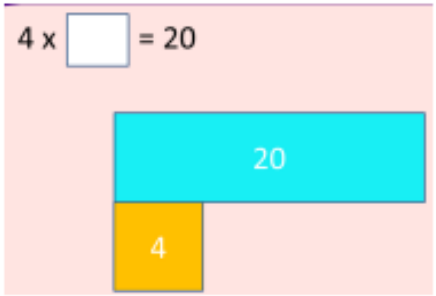
Strategies of teaching: Subtraction

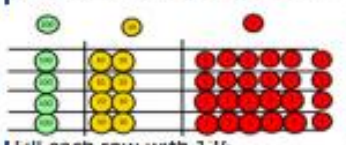
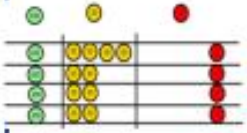
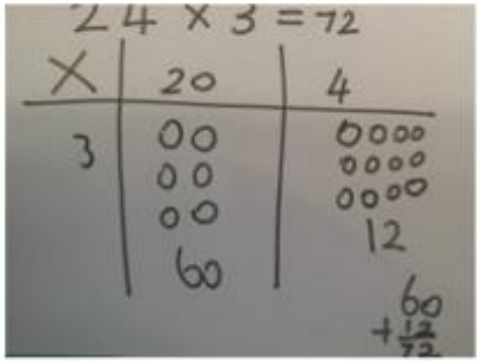
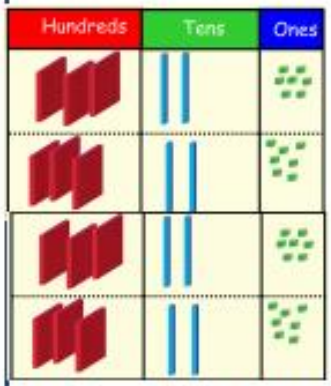
Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers) Y3	 Use base 10 or Numicon to model	 Darw representations to support understanding	 Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method.

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for placeholders. 
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			 

Strategies of teaching: Multiplication

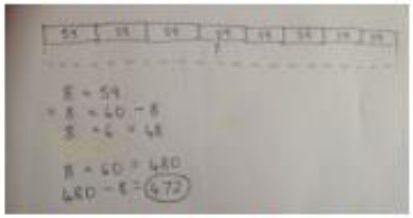
Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative Y3	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ Show all 8 related fact family sentences.

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method Y4	Show the links with arrays to first introduce the grid method  4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed  Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
---	---	---
7	210	35 | 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method. |	10	8
---	---	---
10	100	80
3	30	24	 Then you have your answer.

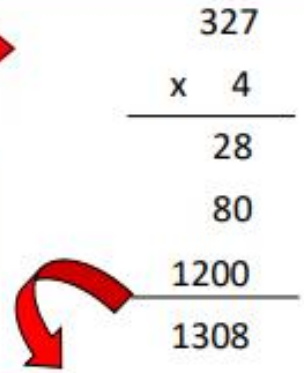
Objective & Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126   Add up each colu…ies making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
---	---	---
7	210	35	 210 + 35 = 245
Column multiplication Y4/Y5	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside		

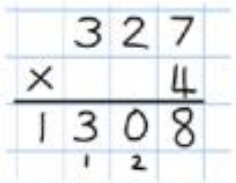
×	300	20	7
4	1200	80	28

The grid method my be used to show how this relates to a formal written method.



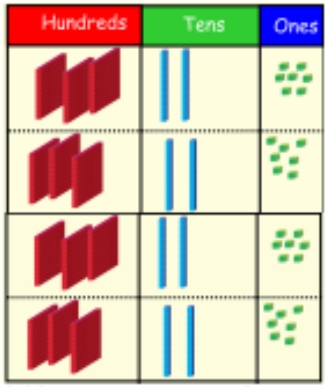
Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

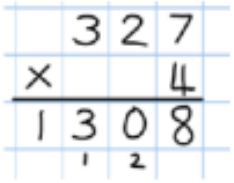




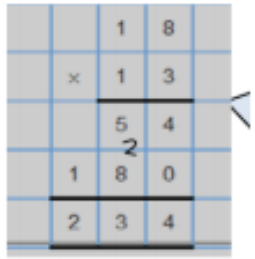
This may lead to a compact method.

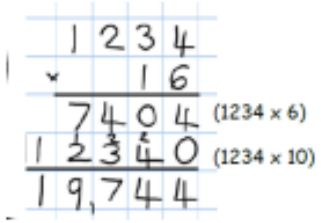
<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Column Multiplication for 3 and 4 digits x 1 digit.

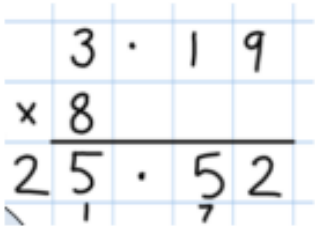
Y5</td>
<td> 

It is important at this stage that they always multiply the ones first.
Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642</td>
<td><table>
<tr><td>x</td><td>300</td><td>20</td><td>7</td></tr>
<tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr>
</table></td>
<td>327
x 4
28
80
1200
1308
 

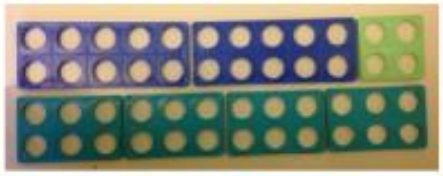
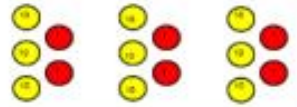
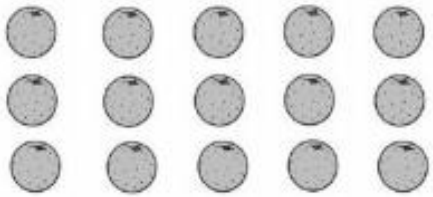
This will lead to a compact method.</td>
</tr>
<tr>
<td>Column multiplication</td>
<td>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</td>
<td> 

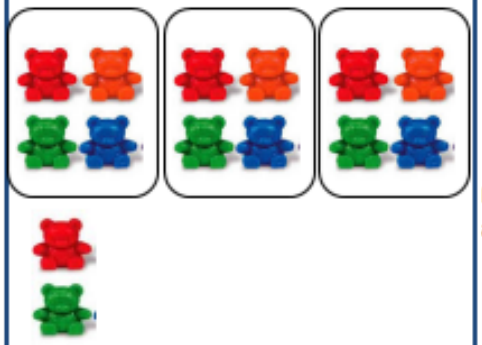
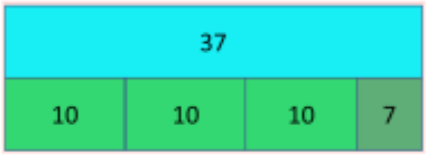
Continue to use bar modelling to support problem solving</td>
<td> 

18 x 3 on the first row
(8 x 3 =24, carrying the 2 for 20, then 1 x 3)
18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first
 
</td>
</tr>
</table>

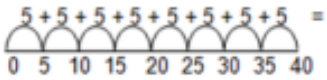
Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit. Y6			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

Strategies of teaching: Division

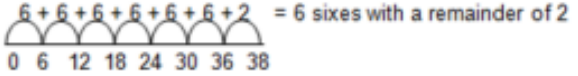
Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping Y3	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders. Y4	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r. 

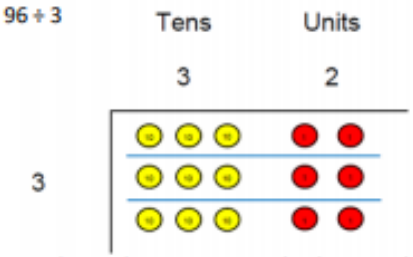
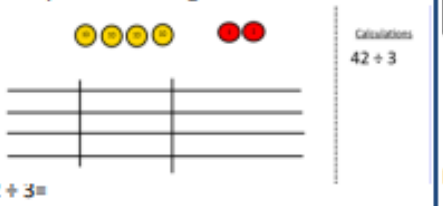
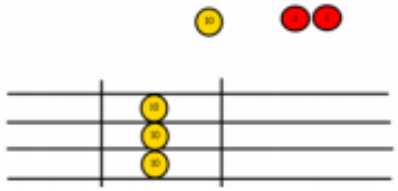
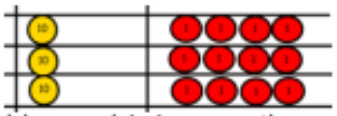
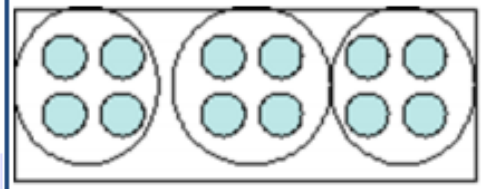
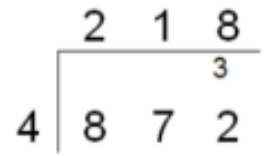
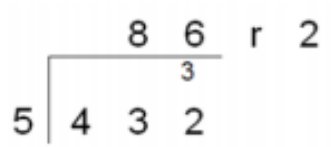
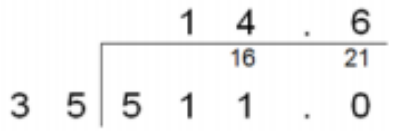
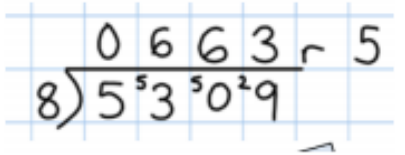
Example without remainder:
40 ÷ 5
Ask "How many 5s in 40?"



Example with remainder:
38 ÷ 6



For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division Y4/Y5	96 ÷ 3  Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  

Y6

Long Division

Step 1—a remainder in the ones

```
    h  t  o
       0  4  1 R1
    ---------
4 ) 1  6  5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
    th h  t  o
       0  4  0  0 R7
    ------------
8 ) 3  2  0  7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

Step 1 continued...

```
    h  t  o
       0  6  1
    ------------
4 ) 2  4  7
      - 4
    ------
         3
```

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

```
    th  h  t  o
        0  4  0  2
    ---------------
4 ) 1  6  0  9
      - 8
    ------
         1
```

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)2 7 8 -2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 1 8 2)2 7 8 -2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 -2 0 7 Divide 2 into 7. Place 3 into the quotient.	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 Divide 2 into 18. Place 9 into the quotient.	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0 There are no more digits to drop down. The quotient is 139.